



$$\int\limits_{dr}^{\mathbb{R}_+} r \gamma \log r^n$$

$$\begin{aligned} \begin{cases} z = \exp(it) r \\ 0 < t < 2\pi \end{cases} \Rightarrow \begin{cases} -z = \exp(i(t-\pi)) r \\ -\pi < t-\pi < \pi \end{cases} \Rightarrow \log(-z) = \log r + i(t-\pi) \\ \overline{\frac{2}{\log(-z)}} = \log^2 r + \overline{t \frac{2}{\pi}} \leq N \log^2 r \\ \overline{^z \gamma z^2} \leq M \Rightarrow \overline{\int_{dz/2\pi}^{\exp(\varepsilon i) R | \exp(-\varepsilon i) R} {}^z \gamma \log(-z)} \leq RN \log R \frac{M}{R^2} = MN \frac{\log R}{R} \underset{R \rightarrow \infty}{\rightsquigarrow} 0 \\ \overline{{}^z \gamma} \leq M \Rightarrow \overline{\int_{dz/2\pi}^{\exp(\varepsilon i) \varrho | \exp(-\varepsilon i) \varrho} {}^z \gamma \log(-z)} \leq \varrho N \log(1/\varrho) = N \frac{\log(1/\varrho)}{1/\varrho} \underset{\varrho \rightarrow 0}{\rightsquigarrow} 0 \\ dz = \exp(it) dr \end{aligned}$$

$$\begin{aligned} \log(-z) = \log r + i(t-\pi) \underset{t \nearrow 2\pi}{\overset{t \searrow 0}{\rightsquigarrow}} \begin{cases} \log r - \pi i \\ \log r + \pi i \end{cases} \Rightarrow \int \log(-z) {}^z \gamma \rightsquigarrow -2\pi i \int\limits_{dr}^{\mathbb{R}_+} r \gamma \\ \log(-z)^2 \rightsquigarrow \begin{cases} \log^2 r - \pi^2 - 2\pi i \log r & t \searrow 0 \\ \log^2 r - \pi^2 + 2\pi i \log r & t \nearrow 2\pi \end{cases} \Rightarrow \int dz {}^z \gamma \log^2(-z) \rightsquigarrow -2\pi i \int\limits_{dr/\pi}^{\mathbb{R}_+} r \gamma \log r \end{aligned}$$

$$\int\limits_{dr/\pi}^{\mathbb{R}_+} \frac{\log r}{(1+r)^3} = -\frac{1}{2}$$

$$\int\limits_{dr/\pi}^{\mathbb{R}_+} \frac{\log r}{(1+r^2)^2} = -\frac{1}{4}$$

$$\int\limits_{dr/\pi}^{\mathbb{R}_+} \frac{\log r}{(1+r^2)(4+r^2)}$$

$$\int\limits_{dr/\pi}^{\mathbb{R}_+}\left\{\begin{matrix}\frac{\log r}{1+r^2}=0\\\frac{\log^3r}{1+r^2}=0\\\frac{\log^2r}{1+r^2}=\frac{\pi^2}{8}\\\frac{\log^4r}{1+r^2}\end{matrix}\right.$$